universität freiburg

Probability Theory **Summer semester 2024**

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https://pfaffelh.github.io/hp/2024ss_wtheorie.html https://www.stochastik.uni-freiburg.de/

Tutorial 1 - Review of measure theory

Exercise 1.
Let Ω be a finite set such that $|\Omega| \geq 4$ and even. Set

$$\mathcal{D} := \{D \subset \Omega \mid |D| \in 2\mathbb{N}\}.$$

Show that $\mathcal{D}$ is a Dynkin system, but not a σ-algebra.

Exercise 2.
Let μ^* be an outer measure on Ω.

1. Prove that if $\mu^*(A) = 0$, then $\mu^*(A \cup B) = \mu^*(B)$.
2. Let (Ω, r) be a metric space, and μ^* the outer measure from Proposition 2.15, where $\mathcal{F}$ is the topology generated from (Ω, r). In addition, let A and B be bounded sets for which there is an $\alpha > 0$ such that $r(a,b) \geq \alpha$ for all $a \in A, b \in B$. Prove that $\mu^*(A \cup B) = \mu^*(A) + \mu^*(B)$.

Exercise 3.
Let μ^* an outer measure on Ω.

1. Let (Ω, r) is a metric space. Show that if a set $E \subseteq \Omega$ has positive outer measure, then there is a bounded subset of E that also has positive outer measure.
2. Show that if E_1 and E_2 are measurable, then

$$\mu^*(E_1 \cup E_2) + \mu^*(E_1 \cap E_2) = \mu^*(E_1) + \mu^*(E_2).$$

Exercise 4.

1. Prove that the set of all real numbers which do not have a 6 in their decimal representation, is a Lebesgue 0-set.
2. Randomly choose an independent and identically, uniformly distributed sequence of numbers in $\{0, \ldots, 9\}$. Compute the probability that the first n numbers are not 6.
3. Do you see a connection between (a) and (b)?