

The background of the slide is a solid blue color with a large, faint watermark of the University of Bonn seal. The seal features a central figure, likely a scholar or saint, surrounded by Latin text and various heraldic symbols.

Measure Theory for Probabilists

1. Introduction

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Introduction

- ▶ Course in spring 2024 at the University of Freiburg
- ▶ All course materials online at
- ▶ Prerequisites: a course in basic probability (coin tossing, throwing dice, binomial distribution, normal distribution)
- ▶ Goal: Solid introduction to all modern probability theory, including weak limits, stochastic processes, etc.
- ▶ Interference: courses in advanced calculus (Analysis III) might also cover measure theory
- ▶ Next course: Probability theory (summer 2024), covering all forms of convergence of random variables, conditional expectation, martingales

Measure theory

- ▶ Sample space Ω ; $A \subseteq \Omega$
- ▶ Assign some value $\mu(A) \in \mathbb{R}_+$ to as many subsets of A as possible, with a number of computation rules
 \Rightarrow measure μ defined on a σ -algebra $\mathcal{F} \subseteq 2^\Omega$
 \rightarrow 1. Set systems; 2. Set functions
- ▶ Make a weighted average of some $f : \Omega \rightarrow \mathbb{R}$ with respect to the measure μ .
 \Rightarrow integral $\int f d\mu$
Study the structure of the space of functions with finite integral
 \rightarrow 3. Measurable functions and the integral; 4. \mathcal{L}^p -spaces
- ▶ All the same on product spaces $\Omega = \times_{i \in I} \Omega_i$
 \rightarrow 5. Product spaces