universität freiburg

Measure theory for probabilists

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https://pfaffelh.github.io/hp/2024WS_measure_theory.html
https://www.stochastik.uni-freiburg.de/

Tutorial 5 - Set systems II

Exercise 1 (4 Points).
Let $\mathcal{C} \subseteq 2^\Omega$. Show that $\mathcal{C} \subseteq \sigma(\mathcal{C})$

Exercise 2 (4 Points).
Show that $\lambda(\mathcal{C})$ is a Dynkin-system.

Exercise 3 (4 Points).

Let $\Omega = \{1,...,n\}$ for some even $n \in \mathbb{N}$ and $\mathcal{D}$ be the set of subsets of even cardinality. Show that $\mathcal{D}$ is a Dynkin system, but it is not a σ-algebra.

Exercise 4 (3+1= 4 Points).

(a) Prove that the intersection of rings is a ring and the intersection of σ-fields is a σ-field. Does the same hold for semi-rings/topologies?

(b) Give a counterexample that shows that, in general, the union of two σ-fields is not necessarily a σ-field.