universität freiburg

Measure theory for probabilists — Winter semester 2024

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https://pfaffelh.github.io/hp/2024WS_measure_theory.html
https://www.stochastik.uni-freiburg.de/

Tutorial 4 - Set systems I

Exercise 1 (4 Points)**.**
Let $\Omega = \{1, \dots, 5\}$. Determine the generated σ-algebra of:

(a) $\mathcal{E} := \{\{1,2,3,4\}\}$,

(b) $\mathcal{F} := \{\{1,2,3\},\{4\}\}$,

(c) $\mathcal{G} := \{1,2,3,4\}$,

(d) $\mathcal{H} := \emptyset$.

Exercise 2 (2+2=4 Points)**.**
Let $f : X \to Y$ be a mapping and let $\mathcal{A} \subset \mathcal{P}(X)$, $\mathcal{B} \subset \mathcal{P}(Y)$. Decide (with reasons) whether the following statements are correct:

(a) If $\mathcal{A}$ is a σ-algebra, then $\{B \subset Y \mid f^{-1}(B) \in \mathcal{A}\}$ is also a σ-algebra.

(b) If $\mathcal{B}$ is a σ-algebra, then $f^{-1}(\mathcal{B}) := \{f^{-1}(B) \mid B \in \mathcal{B}\}$ is also a σ-algebra.

Exercise 3 (4 Points)**.**
Let Ω be an uncountable set and

$$\mathcal{A} = \{A \subset \Omega \mid A \text{ or } A^c \text{ is countable}\}.$$

Show that $\mathcal{A}$ is a σ-algebra.

Exercise 4 (4 Points)**.**
Find an example of a Dynkin system that is not a semiring.