universität freiburg

Measure theory for probabilists — Winter semester 2024

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https://pfaffelh.github.io/hp/2024WS_measure_theory.html
https://www.stochastik.uni-freiburg.de/

Tutorial 3 - Further review of toplogy

Exercise 1 (4 Points).
Show that there exists a sequence of open sets $\{O_n\}_{n\in\mathbb{N}} \subset \mathcal{O}$ such that $\bigcap_{n\in\mathbb{N}} O_n$ is not an open set.

Exercise 2 (4 Points).
Let $x \in \mathbb{R}^n$, and let $\epsilon > 0$. Let $y \in \mathcal{B}_\epsilon(w)$. Show that

$$\mathcal{B}_{\epsilon-||w-y||}(w) \subset \mathcal{B}_\epsilon(w).$$

Exercise 3 (4 Points).
Let x,$y \in \mathbb{R}^n$ and $r = ||x - y||$. Show that

$$\mathcal{B}_{\frac{r}{2}}\left(\frac{x+y}{2}\right) \subset \mathcal{B}_r(x) \cap \mathcal{B}_r(y).$$

Exercise 4 (4 Points).
Let (X,r) and (Y,r') be metric spaces and $f : X \to Y$. Show that f is continuous on X if and only if for every closed set A in Y, $f^{-1}(A)$ is closed in X. See Defintion A.1(10) for the general definition on topological spaces.