universität freiburg

Measure theory for probabilists **Winter semester 2024**

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https://pfaffelh.github.io/hp/2024WS_measure_theory.html
https://www.stochastik.uni-freiburg.de/

Tutorial 2 - Review of topology and compactness

Exercise 1.
Let $X = \{a,b,c,d\}$. Which of the following are topologies for X?

(i) $\{\emptyset, X, \{a\}, \{b\}, \{a,c\}, \{a,b,c\}, \{a,b\}\}$

(ii) $\{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{b,d\}\}$

(iii) $\{\emptyset, X, \{a,c,d\}, \{b,c,d\}\}$

Can you further give an example of two subsets A and B of $\mathbb{R}$ such that

$$A \cap B = \emptyset, \quad \overline{A} \cap B \neq \emptyset, \quad A \cap \overline{B} \neq \emptyset.$$

Exercise 2.
Prove that a subset of a topological space (Ω, r) is open if and only if its complement in Ω is closed.

Exercise 3 (4 Points)**.**
Let A and B be compact subsets of a metric space (X, r). Show that $A \cap B$ and $A \cup B$ are also compact.

Exercise 4 (4 Points)**.**
Consider the cofinite topology $\mathcal{O}$ on $\mathbb{Z}$ defined as follows: a subset $O \subset \mathbb{Z}$ is an open set if and only if $O = \emptyset$ or $O = O^c$ is finite. Show that $\mathcal{O}$ is a topology in $\mathbb{Z}$.