universität freiburg

Probability Theory — Summer semester 2024

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https://pfaffelh.github.io/hp/2024ss_wtheorie.html
https://www.stochastik.uni-freiburg.de/

Tutorial 3 - Random variables and characteristic functions

Exercise 1 (4 Points).
Justify whether or not the following statements are true.

(a) Let $X \sim \mathcal{N}(0,1)$ and $Y \sim \mathcal{N}(0,2)$ be normally distributed. Then $E[XY] \leq \sqrt{2}$.

(b) Let X be exponentially distributed with respect to the parameter $\lambda = 6$ and Y exponentially distributed with respect to the parameter $\lambda = \frac{1}{3}$. Then

$$E[XY] \leq 1.$$

(c) Let X be exponentially distributed with parameter $\lambda = 1$. Then $E[X^4] \geq E[X]^4$.

Exercise 2 (4 Points).
Let $X \sim \text{Poi}(\gamma)$ and $Z \sim \mathcal{N}(0,1)$. Show that for all $t \in \mathbb{R}$

$$\psi_{\frac{X-\gamma}{\sqrt{\gamma}}}(t) \xrightarrow{\gamma \to \infty} \psi_Z(t).$$

Exercise 3 (4 Points).
Let $X, Y \sim \exp(1)$ be independent and $U \sim U([0,1])$. Show that $X/(X+Y) \sim U$.

Exercise 4 (4 Points).
Let $p, q > 0$ and

$$g_{p,q}(x) := x^{p-1}(1-x)^{q-1}.$$

(a) Show that

$$B(p,q) := \int_0^1 g_{p,q}(x)dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)},$$

where Γ is the Gamma-function.

(b) Let $X_{p,q}$ be a random variable with density $f(x) = \frac{1}{B(p,q)} g(x) 1_{0 \leq x \leq 1}$. Show that $X_{q,p} \sim 1 - X_{p,q}$.